

# Improved DMC Design for Nonlinear Process Control

Chi Min Chang, San Jang Wang, and Shuh Woei Yu

Dept. of Chemical Engineering, National Central University, Chungli, Taiwan, ROC

Modeling philosophy and the unique ability of dynamic matrix control (DMC) in handling complex control problems commonly encountered in multivariable systems such as process constraints, interactions, and delays have made it a very popular control algorithm in the process industry. Basic concepts and application examples of DMC were presented by many associated with its early development (Cutler and Ramaker, 1979; Prett and Gillette, 1979; Cutler, 1983; Cutler and Hawkins, 1988; Prett and Garcia, 1988). This prediction- and optimization-based control strategy has attracted a fair amount of academic research interest as well. Marchetti et al. (1983), using a number of single-loop examples, studied the effects of various design parameters on the performance of DMC. A theoretical analysis of DMC, by drawing its parallels in statistical concepts, provided another vehicle in understanding its success in industrial applications (Ogunnaike, 1986). Recently, Maurath et al. (1988) proposed a principal components analysis based on the singular value decomposition (SVD) of the constructed dynamic matrix. Impact of the number of components retained in the SVD-based least-square solution on performance and control energy was investigated to facilitate the design of DMC controllers.

The predictive ability of DMC, although effective in controlling linear multivariable systems, can face serious design and implementation problems in nonlinear processes due to convolution model variations caused by asymmetric effects of the process input variables. Using moderate- and high-purity distillation towers, McDonald and McAvoy (1987) illustrated the difficulty in obtaining a representative process model for nonlinear systems and proposed a gain and time constant scheduling technique to update the dynamic matrix to improve control performance. However, the computational requirement for on-line process parameter evaluation and updating, even for the first order with delay processes and on-line matrix inversion, may limit its applicability. To reduce or eliminate the apparent nonlinearity of the process and improve control performance, Georgiou et al. (1988) proposed the use of variable transformation in deriving the so-called nonlinear DMC (NLDMC). Open-loop step responses of the transfer-function-

based nominal models were used to construct the convolution model in comparing the performance of standard DMC and nonlinear DMC.

Although nonlinear physical models were used in both studies mentioned above, emphasis was put on empirically updating the process model without redesign of the DMC controllers in the first study. Georgiou and coworkers (1988), on the other hand, concentrated on the DMC design using step responses of the nominal models as convolution models. Impact of the convolution model derived from the rigorous simulation model on controller design or performance was not considered. The concept of an average convolution model is introduced in this study to enhance the design of DMC controllers, standard or nonlinear, for dual-composition control of a high-purity distillation column. Impact of input perturbation sizes on control performance is also investigated.

## Column Characteristics and DMC Design

The rigorous tray-by-tray simulation model of Chiang (1985) was adopted in this study. This high-purity column was designed for methanol/water separation with overhead and bottoms product purity set at 99.9 mol % and 0.1 mol %, respectively. Nominal models of the column and the variable transformed system with  $\bar{X}_D = \ln[(1 - X_D)/(1 - X_D^{set})]$  and  $\bar{X}_B = \ln(X_B/X_B^{set})$  are directly taken from Georgiou et al. (1988) and listed in Table 1. For nonlinear processes such as this

**Table 1. Nominal Models for the Standard and Transformed Systems**

$\begin{pmatrix} X_D \\ X_B \end{pmatrix} = \begin{pmatrix} \frac{0.71e^{-6.5s}}{(1+117s)(1+3.5s)} & \frac{-0.91e^{-6.3s}}{(1+152s)(1+2.4s)} \\ \frac{7.1e^{-11.5s}}{(1+246s)(1+1.3s)} & \frac{-14.5e^{-6.2s}}{(1+208s)(1+0.8s)} \end{pmatrix} \begin{pmatrix} L \\ V \end{pmatrix}$		
$\begin{pmatrix} \bar{X}_D \\ \bar{X}_B \end{pmatrix} = \begin{pmatrix} \frac{-56.5e^{-6.5s}}{(1+120s)(1+3.4s)} & \frac{73e^{-6.3s}}{(1+130s)(1+3.6s)} \\ \frac{564e^{-11.5s}}{(1+222s)(1+1.5s)} & \frac{-1,162e^{-6.2s}}{(1+181s)(1+0.7s)} \end{pmatrix} \begin{pmatrix} L \\ V \end{pmatrix}$		

Correspondence concerning this work should be addressed to Shuh Woei Yu.

particular column, open-loop step responses needed to construct the convolution model of the DMC controller depend heavily on the size and direction of the input step. In addition, the asymmetric dynamic behavior of the process occurring at different operating points must be considered to obtain a more representative convolution model. The first problem can usually be resolved with relatively small, for instance, less than  $1 \times 10^{-4}$ , perturbations in the manipulated variables. As for the second concern, the concept of an average convolution model is proposed in this study to capture not just the local dynamics of the process at any point. And it will be shown that utilization of the average model can greatly improve performance of the DMC controllers.

For consistency, the operating range of the overhead composition was chosen to be from 0.9992 to 0.996 mol fraction with the bottoms composition fixed at 0.001 mol fraction to match the servo control conditions specified by Georgiou et al. (1988). A steady-state design package with varying  $X_D$ , by dividing the selected range of 0.9992 to 0.996 mol fraction into six discrete segments and fixed  $X_B$  at 0.001 mol fraction, is used to generate the initial column profiles needed to execute the dynamic responses. An arithmetic mean of the step response data, obtained at each operating point by making a 0.01% perturbation in reflux and boilup rate, respectively, is then calculated to construct the average convolution model. Without variable transformation and using the final steady-state gains of Table 1 as reference, the  $X_D-L$  gain of the average convolution model is about twice as that of the nominal model, whereas the  $X_D-V$  gain decreases by about 300%. Mismatch of the bottoms loop is much less severe. The  $X_B-L$  gain increases about 28%, and  $X_B-V$  gain of the convolution model decreases by 30%. In the case of variable transformation, plant/model mismatch of the overhead loop is greatly reduced. Relative to the nominal steady-state process gains, the  $\bar{X}_D-L$  gain decreases by about 40%, while the  $\bar{X}_D-V$  gain increases by 77%. The bottoms loop possesses almost the same amount of mismatches as in the previous case. This serves to prove that certain amount of nonlinearity can indeed be reduced via variable transformation.

With the average convolution model determined from the simulation results, one can then proceed with the remaining design steps of DMC. In this study, the sampling period ( $T_s$ ) is also chosen to be six minutes. The products of  $T_s$  times the number of coefficients in the convolution model ( $N_s$ ) and the number of future moves ( $N_c$ ) are set to be equal to the time required for the slowest open-loop response to reach 95% and 60% of the final steady-state value, respectively, as recommended by Cutler (1983).

The last design step is then to determine the input move suppression factors,  $K_1$  and  $K_2$ . The principal components analysis method is not considered in this study, because it may involve computational efforts. To thoroughly investigate the impact of convolution model on servo control of DMC, Rosenbrock's multivariable constrained optimization routine (Kuester and Mize, 1973) is utilized to search for the optimal  $K$  values. For a fair comparison with the controllers designed by Georgiou et al. (1988), the optimization procedure uses the integral of absolute errors (IAE) of both loops as the objective function subject to the same allowable control actions. The nominal linear models in Table 1 are solved to generate the dynamic responses needed in the optimization program. In

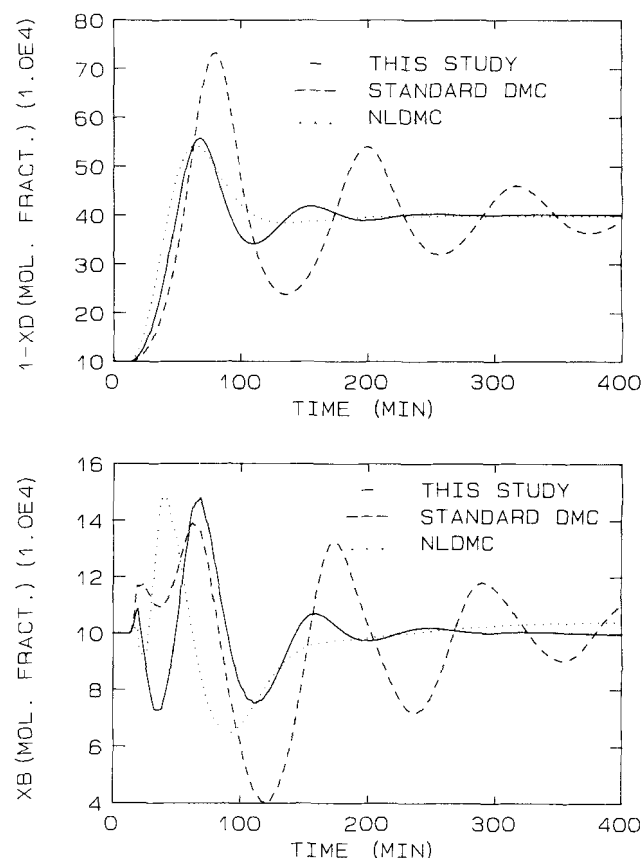
**Table 2. Design Parameters and Performance Comparison**

Controller Type	Convolution Model Basis	$N_s$	$N_c$	$K_1$	$K_2$	IAE	
						$X_D^{\text{set}}$ Decrease	$X_D^{\text{set}}$ Increase
DMC with Std. Nominal Model		150	30	0.7	0.7	0.546	0.023
NLDMC with Transf. Nominal Model		150	30	7.0	7.0	0.208	0.017
DMC with Avg. Convolution Model		153	49	0.35	0.47	0.219	0.019

addition, the same weighting factor is applied to all future outputs.

## Results and Discussion

Servo control, a decrease of 0.3% and an increase of 0.02% in the initial  $X_D$  set point of 0.999 mol fraction, respectively, is used to illustrate performance improvement of the DMC control scheme designed with the average convolution model. Basis of the step response models, DMC design parameters, and the calculated IAE values at the final steady state are given in Table 2. Figure 1 provides the closed-loop rigorous nonlinear dynamic simulation results for a -0.3% step change in the  $X_D$  set point. For comparison, the original design of the nonlinear DMC of Georgiou et al. (1988) is also included. Superiority of the proposed design approach over the nominal-



**Figure 1. Closed-loop responses for  $X_D$  set point change from 0.999 to 0.996.**

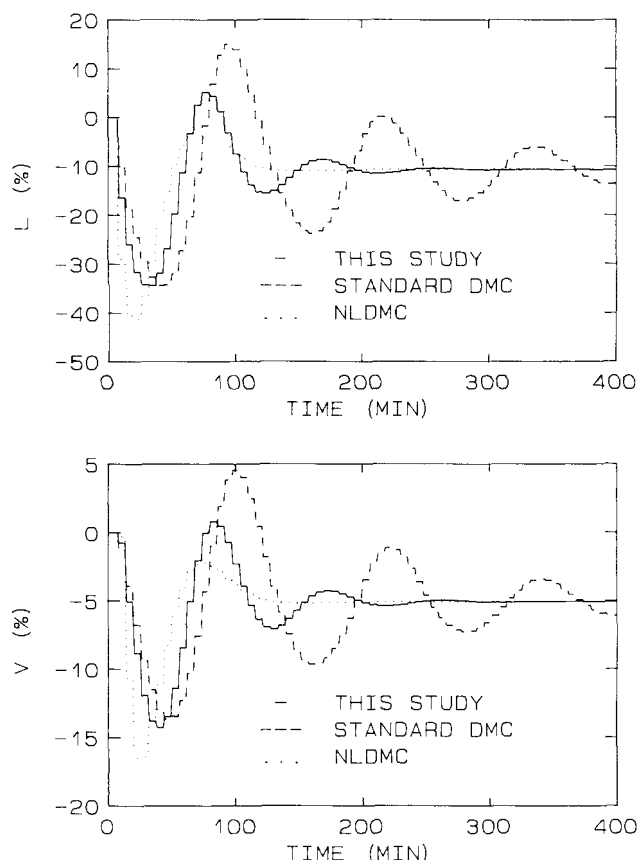


Figure 2. Control actions for a decrease in  $X_p$  set point.

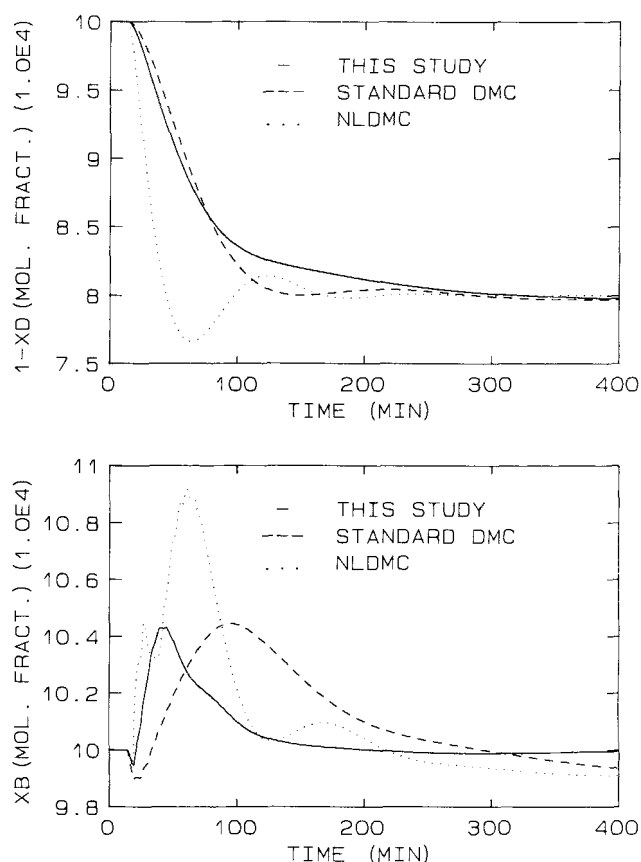


Figure 3. Closed-loop responses for  $X_p$  set point change from 0.999 to 0.9992.

model-based DMC controllers is clearly demonstrated. It is also significant to note that the standard DMC designed with the average convolution model is capable of providing the same control performance as that of the nonlinear DMC. Comparable adjustments in  $L$  and  $V$  by all three control schemes are observed as shown in Figure 2. Figure 3 shows the column responses subject to a 0.02% increase in overhead product purity. Notable improvement in the bottoms control loop is achieved by the proposed design technique, whereas the NLDMC provides the best control of the overhead loop. However, control actions required by NLDMC controllers are much more excessive than other designs as shown in Figure 4.

The average convolution model concept is further utilized to analyze the impact of convolution models on the control performance of standard DMC and nonlinear DMC controllers. This is conducted by using different sizes of manipulated variable perturbations,  $1 \times 10^{-4}$  and  $1 \times 10^{-6}$ , respectively, to generate the simulation data. Table 3 summarizes the controller parameters and IAEs for each design. In general, differences in process gains between the different average convolution models are in the neighborhood of few percentage points in both versions of DMC. Different values of  $N_s$  and  $N_c$  in Table 3 can be viewed as variations in response time. An advantage of the standard DMC approach is that its performance is less sensitive to the magnitude of the manipulated variable perturbations. However, the convolution model based on smaller perturbations results in some improvement in the nonlinear DMC approach as the IAE values are reduced by about 30%. It seems that small input perturbations are preferable in gen-

erating the dynamic matrix for nonlinear DMC implementation.

The concept of average convolution model is introduced in this study to enhance the control of nonlinear processes of DMC. Simulation results of a rigorous nonlinear distillation model clearly demonstrate the potential of the proposed design technique. In industrial applications, however, a couple of issues concerning the construction of the average convolution model have to be addressed. The first problem encountered in the process industry is the lack of accurate simulation models that can be utilized for the design and analysis of control algorithms such as the proposed DMC approach. And the alternative of conducting step tests with varying input magnitude and direction on a real process at different operating conditions to generate the convolution coefficients may also be infeasible. Fortunately, both problems can be alleviated with the advanced nonlinear process identification techniques such as autotune (Luyben, 1987) and the improved autotune (Li et al., 1991). The second method is extremely attractive for real industrial applications, since it requires no prior knowledge of the steady-state process gain and hence reduces the number of test runs. Since process transfer functions are used in most controller design methodologies, the linear models derived with these techniques at different operating conditions can be utilized to achieve better understanding of the nonlinear behavior of the process and generate the data required by the average convolution model.

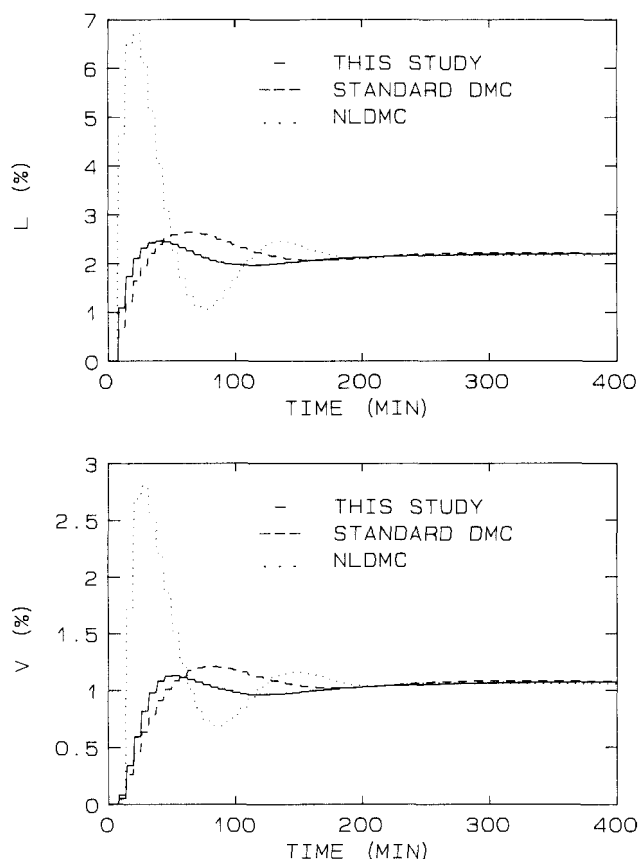


Figure 4. Control actions for an increase in  $X_D$  set point.

## Notation

$K_i$  = suppression factor or DMC tuning parameters ( $i=1,2$ )  
 $L$  = reflux rate (mol/min)  
 $N_c$  = number of future moves  
 $N_s$  = number of coefficients of convolution model  
 $T_s$  = sampling period (min)  
 $V$  = vapor boilup rate (mol/min)  
 $X_B$  = bottom composition (mol fraction)  
 $\bar{X}_B$  = nonlinear output transformation (bottom)  
 $X_B^{set}$  = set point of bottom composition (mol fraction)  
 $X_D$  = top composition (mol fraction)  
 $\bar{X}_D$  = nonlinear output transformation (top)  
 $X_D^{set}$  = set point of top composition (mol fraction)

## Literature Cited

Chiang, T. P., "Dynamics and Control of Heat Integrated Distillation Columns," *PhD Thesis*, Lehigh Univ. (1985).

Table 3. Impact of Input Step Sizes on Control System Performance

Controller Type Perturbation Size	$N_s$	$N_c$	$K_1$	$K_2$	IAE	
					$X_D^{set}$ Decrease	$X_D^{set}$ Increase
DMC with Perturb. of $1 \times 10^{-4}$	153	49	0.35	0.47	0.219	0.019
DMC with Perturb. of $1 \times 10^{-6}$	144	47	0.35	0.36	0.210	0.020
NLDMC with Perturb. of $1 \times 10^{-4}$	147	45	4.57	9.25	0.174	0.013
NLDMC with Perturb. of $1 \times 10^{-6}$	138	45	4.31	8.70	0.139	0.010

- Cutler, C. R., "Dynamic Matrix Control: an Optimal Multivariable Control Algorithm with Constraints," *PhD Thesis*, Univ. of Houston (1983).
- Cutler, C. R., and R. B. Hawkins, "Application of a Large Predictive Multivariable Controller to a Hydrocracker Second Stage Reactor," *Proc. Am. Control Conf.*, Atlanta, 284 (1988).
- Cutler, C. R., and B. L. Ramaker, "Dynamic Matrix Control—a Computer Control Algorithm," *AICHE Meeting*, Houston (1979).
- Georgiou, A., C. Georgakis, and W. L. Luyben, "Nonlinear Dynamic Matrix Control for High-Purity Distillation Columns," *AICHE J.*, **34**, 1287 (1988).
- Kuester, J. L., and J. H. Mize, *Optimization Techniques with Fortran*, McGraw-Hill, New York (1973).
- Li, W., E. Eskinat, and W. L. Luyben, "An Improved Autotune Identification Method," *Ind. Eng. Chem. Res.*, **30**, 1530 (1991).
- Luyben, W. L., "Derivation of Transfer Functions for Highly Nonlinear Distillation Columns," *Ind. Eng. Chem. Res.*, **26**, 2490 (1987).
- Marchetti, J. L., D. A. Mellichamp, and D. E. Seborg, "Predictive Control Based on Discrete Convolution Models," *Ind. Eng. Chem. Process Des. Dev.*, **22**, 488 (1983).
- Maurath, P. R., A. J. Laub, D. E. Seborg, and D. A. Mellichamp, "Predictive Control Design by Principal Components Analysis," *Ind. Eng. Chem. Res.*, **27**, 1204 (1988).
- McDonald, K. A., and T. J. McAvoy, "Application of Dynamic Matrix Control to Moderate- and High-Purity Distillation Towers," *Ind. Eng. Chem. Res.*, **26**, 1011 (1987).
- Ogunnaike, B. A., "Dynamic Matrix Control: a Nonstochastic, Industrial Process Control Technique with Parallels in Applied Statistics," *Ind. Eng. Chem. Fundam.*, **25**, 712 (1986).
- Prett, D. M., and C. E. Garcia, *Fundamental Process Control*, Butterworths, Stoneham, MA (1988).
- Prett, D. M., and R. D. Gillette, "Optimization and Constrained Multivariable Control of a Catalytic Cracking Unit," *AICHE Meeting*, Houston (1979).

Manuscript received Sept. 9, 1991, and revision received Dec. 9, 1991.